Exploring a Two Population Genetic Algorithm

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GAs and Constrained Optimization

- Constrained optimization is difficult: genetic operators don't respect feasibility.
- Approaches to handling infeasible solutions: discard (wasteful), penalty functions (but which?), special encoding to guarantee feasibility (rarely available), repair of infeasible solutions (problem-specific; computationally costly, often).
- Why bother with GAs? Some metaheuristic is often required. GAs are easy to use.
- Elsewhere: GAs leave useful information as a side-effect (candle-lighting analysis).

Our New Idea: Separate Feasibility and Optimality Evaluations

- GECCO 2002: Two market GA. Excellent results on knapsack problems, tested against standard penalty methods.
- But: knapsack, while NP-hard, is thought to be atypically easy.
- But: there are standard GA packages out there for comparison.
- And: an improvement, refinement, on the concept: two population GA.

GECCO 2003 Paper

- Beyond knapsack to nonlinear and mixed integer problems.
- Comparison with performance of a standard GA package, Genocop II/III.
- Refined concept: two population GA.

  Feasible population. Infeasible population. Evaluation alternates. Children are placed where they belong.

Group 1 Problems, from Michalewicz

Problem	Min/Max	Objective	# Variables	# Linear Inequalities	# Nonlinear Inequalities
1	min	Linear	8	3	3
2	min	Polynomial	7	0	4
3	min	Quadratic	10	3	5
4	min	Polynomial	2	0	2
5	min	Linear	2	0	2
6	min	Polynomial	2	0	2
7	min	Quadratic	2	0	2
8	min	Quadratic	2	1	1
9	max	Nonlinear	20	1	1
10	max	Nonlinear	50	1	1

Nonlinear, with floating point variables. Arithmetic crossover, non-uniform mutation.

Group 1 Results: With Untuned 2 Market GA

Prob.	Min/ Max	Best Known or Optimal*	Genocop II/III	Two-Market GA		
				Best of 10	Median	Std.
⇒1	min	7049.330923*	7268.650	∅	∅	∅
2	min	680.6300573*	680.640	680.6374	680.7566	0.085123
⇒3	min	24.3062091*	25.883	25.18437	25.61935	0.825277
4	min	0.25*	close	0.25	0.25	0
5	min	-5.5079*	close	-5.50773	-5.50708	0.001042
6	min	-6961.81381*	close	-6960.95	-6957.44	1.908346
7	min	5*	close	5	5	0
8	min	1*	close	1	1	0
⇒9	max	0.80351067	0.80351067	0.80288	0.7888	0.12853
10	max	0.8348	0.83319378	0.809211	0.743844	0.054728

Group 2 Problems: GLOBAL Library at GAMS World

Problem	Min/Max	Objective	# Variables	# Linear Inequalities	# Nonlinear Inequalities
11 (test11)	max	Nonlinear	2	0	2
12 (chance)	min	Linear	3	2	1
13 (circle)	min	Linear	3	0	10
14 (ex3_1_4)	min	Linear	3	2	1
15 (ex7_3_1)	min	Linear	4	6	1
16 (ex7_3_2)	min	Linear	4	6	1
17 (ex14_1_1)	min	Linear	3	0	4
18 (st_e08)	min	Linear	2	0	2
19 (st_e12)	min	Nonlinear	3	3	0
20 (st_e19)	min	Polynomial	2	1	1
21 (st_e41)	min	Nonlinear	4	0	2

Group 2 Results

Problem	Best Known or Optimal*	Genocop III		Two-Population GA	
		Best of 10	Std.	Best of 10	Std.
11	?	0.115047	4.87E-05	0.115047	4.16E-08
12	29.8943781591	29.89549	0.034976	†	0.032017
13	4.57424778502	4.574318	0.027615	4.574248	6.67E-05
14	-4.0000	-4	0.032482	-4	0.000131
15	0.341739553124	0.3558	0.141607	‡	0.00033
16	1.08986397147	1.09145	0.033588	1.089952	2.56E-05
17	0.0000	1.44E-10	2.85E-06	4.69E-05	1.36E-05
18	?	0.741782	9.93E-09	0.741782	5.58E-08
19	?	-4.5099	0.02252	-4.51347	0.000995
20	?	-118.705	0.021621	-118.705	0.000187
21	?	645.626	4.398663	641.8244	2.663347

† = 29.8943786178232, ‡ =**0.319777729979837**. NB. untuned, lower variance.

Infeasible Children Decrease over Time

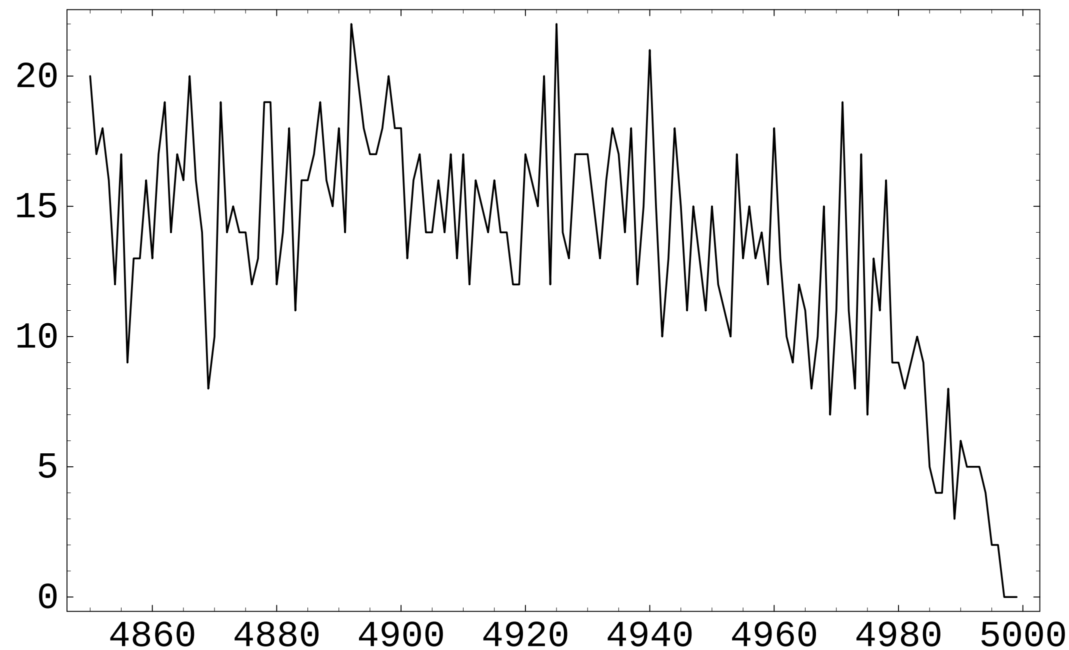

Figure 1: For problem ex7_3_2 with population size 50: count of number of infeasible solutions created from the feasible population, by generation, 4850–4999.

Continuing & Increasing Contribution by the Infeasible

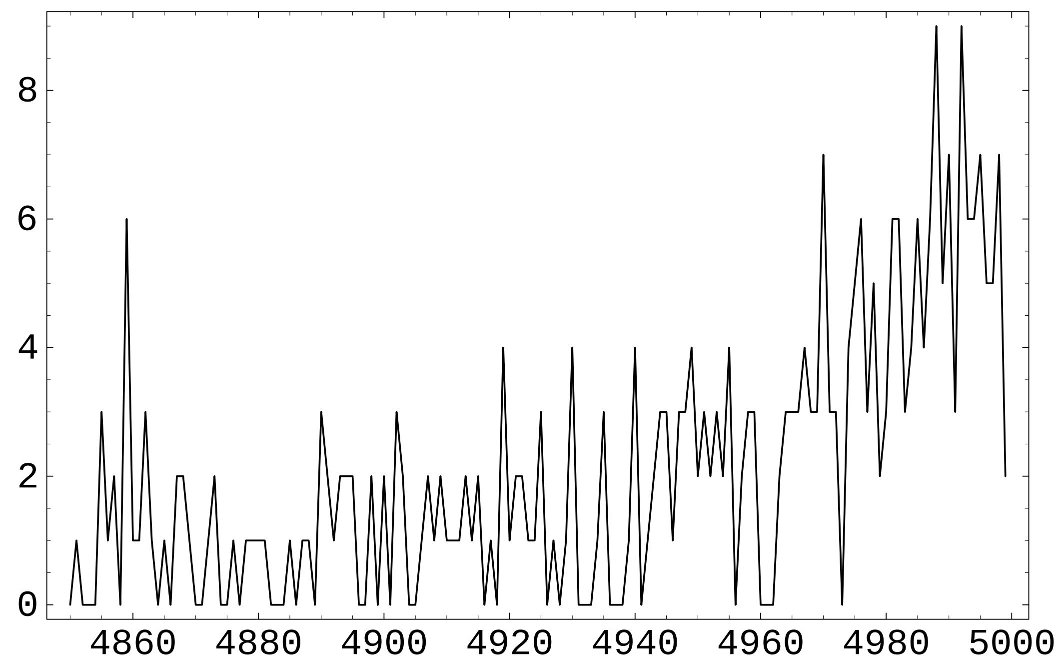

Figure 2: For problem ex7_3_2 with population size 50: count of number of feasible solutions created from the infeasible population, by generation, 4850–4999.

Everyone Is Descended from a Loser

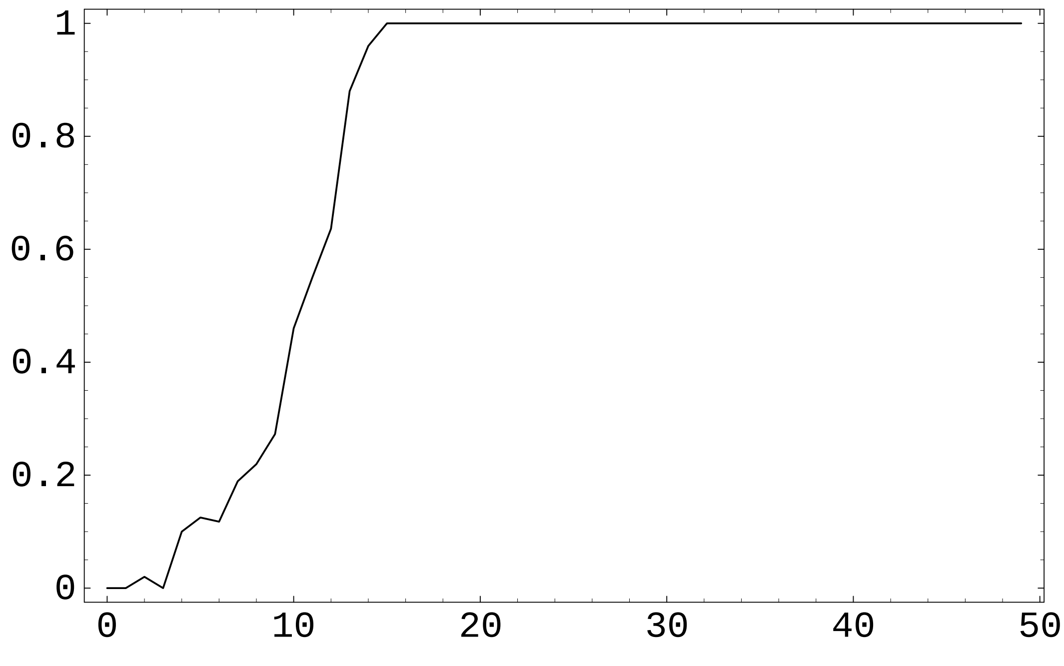

Figure 3: For problem ex7_3_2 with population size 50: fraction, by generation, of feasible individuals with an infeasible ancestor.

Summary: Why/How Does It Work?

- The continuing contributions of losers.
- The infeasible population (after some selection) retains valuable genetic information.
- There is some optimal rate of exchange between having an extra feasible solution and having an extra infeasible solution.

Late Breaking Results: Comparison with PSO

- See: Parsopoulos, K.E., Vrahatis, M.N., "Particle Swarm Optimization Method for Constrained Optimization Problems," in P. Sincak, J. Vascak, V. Kvasnicka, J. Pospichal (eds.), *Intelligent Technologies - Theory and Applications: New Trends in Intelligent Technologies*, pp. 214-220, IOS Press (Frontiers in Artificial Intelligence and Applications series, Vol. 76), 2002, ISBN: 1-58603-256-9. `http://www.math.upatras.gr/~kostasp/papers/eisci.pdf`. And generally, `http://www.math.upatras.gr/~kostasp/public.html`.

- Used PSO with dynamic penalty functions to solve a number of test problems.

- We examined 3 of their 6 problems.

Problem Summary

Test Problem	Min/ Max	Objective Function	# Vars.	# Linear Inequalities	# Nonlinear Inequalities
#4	Min	Quadratic	5	0	6
#5	Min	Quadratic	5	0	6
#6	Min	Quadratic	6	2	0

Problems #4 & #5

$$\min f(x) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141$$

subject to $0 \leq 85.334407 + 0.0056858T_1 + T_2x_1x_4 - 0.0022053x_3x_5 \leq 92$,
$90 \leq 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2 \leq 110$,
$20 \leq 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 \leq 25$,
$78 \leq x_1 \leq 102$, $33 \leq x_2 \leq 45$, $27 \leq x_i \leq 45$, $i = 3, 4, 5$,

where in #4, $T_1 = x_2x_5$ and $T_2 = 0.0006262$ and for #5, $T_1 = x_2x_3, T_2 = 0.00026$. Best known solution for #4 is $f^* = -30665.538$. #5 has an unknown best solution.

Problem #6

$$\min f(x, y) = -10.5x_1 - 7.5x_2 - 3.5x_3 - 2.5x_4 - 1.5x_5 - 10y - 0.5 \sum_{i=1}^{5} x_i^2$$

subject to $6x_1 + 3x_2 + 3x_3 + 2x_4 + x_5 - 6.5 \leq 0$, $10x_1 + 10x_3 + y \leq 20$, $0 \leq x_i \leq 1$, $i = 1, \ldots, 5$, $0 \leq y$.

Best known solution for #6 is $f^* = -213.0$.

Results Compared to Genocop

Settings as in GECCO 2003 paper: population size 50, 10,000 generations, 10 runs each.

Test Problem	Best Known or Optimal	Genocop III		Two Population GA	
		Best	Std.	Best	Std.
#4	-30665.538	-30663.029297	45.381845	-30665.509108	0.019961
#5	?	-30986.876953	35.950656	-31026.424543	0.029210
#6	-213	-212.3456573	3.684304	-213	0

Two-market GA does better in all three cases: best and for std.

PSO (mostly infeasible) vs. feasible 2 pop. GA

Unspecified computational effort. Sum V.C.= sum of violated constraints.
PSO solutions generally not feasible.

Test Problem	Method	Best Solution (Sum V.C.)	Mean (Sum V.C.)	2 Pop GA
#4	PSO-In	-31543.484 (1.311)	-31526.304 (1.297)	-30665.509
	PSO-Co	-31542.578 (1.311)	-31528.289 (1.326)	
	PSO-Bo	-31544.459 (1.311)	-31493.190 (1.331)	
#5	PSO-In	-31544.036 (0.997)	-31523.859 (0.958)	-31026.425
	PSO-Co	-31543.312 (0.996)	-31526.308 (0.965)	
	PSO-Bo	-31545.054 (0.999)	-31525.492 (0.968)	
#6	PSO-In	-213.0 (0.0)	-213.0 (0.0)	-213.0
	PSO-Co	-213.0 (0.0)	-213.0 (0.0)	
	PSO-Bo	-213.0 (0.0)	-213.0 (0.0)	

Set Covering Problems

Th. Bäck, M. Schütz and S. Khuri: "A Comparative Study of a Penalty Function, a Repair Heuristic, and Stochastic Operators with the Set-Covering Problem," J. M. Alliot, E. Lutton, E. Ronald, M. Schoenhauer and D. Snyers, editors: *Artificial Evolution*, pp. 3-20, Springer, Berlin, 1996. `http://ls11-www.cs.uni-dortmund.de/people/baeck/papers/ea95.ps.gz`

Links to Thomas Bäck's website:

`http://ls11-www.cs.uni-dortmund.de/people/baeck/`

`http://ls11-www.cs.uni-dortmund.de/people/baeck/ea_application.html`

(The downloadable files are actually in .ps format, not ps.gz. You need rename files and you can use Adobe Acrobat to convert them to pdf files.)

7 set covering problems from Beasleys OR-library

We apply our two-population GA on 7 set covering problems (all minimizing) from Beasleys OR-library. For detailed information and the data file for these test problems, please refer to the following website: http://mscmga.ms.ic.ac.uk/jeb/orlib/scpinfo.html. m=number of constraints. n=number of (0-1) decision variables.

Problem	m	n	density
Scp41	200	1000	2
Scp42	200	1000	2
Scpe1	50	500	20
Scpe2	50	500	20
Scpe3	50	500	20
Scpe4	50	500	20
Scpe5	50	500	20

Mixture of Results Compared to Bäck et al.

Test Problem	Best-Known /Optimal	Two Population GA			Bäck et al.	
		Best	Avg.	Stdev.	Penalty	Repair
Scp41	429	1069	1385	230.1267	1030	439
Scp42	512	1208	1497	181.1648	1208	536
Scpe1	5	6	7	0.92582	6	8
Scpe2	5	6	6.857143	0.638877	7	7
Scpe3	5	6	7	0.755929	7	7
Scpe4	5	7	7.285714	0.451754	7	6
Scpe5	5	6	6.928571	1.083268	8	7

Computational effort: Bäck et al.: population size = 100, generations=5000. We did 50, 50 at 5,000 each. But, cost of repair?

In sum: remarkably good and robust performance from a simple, untuned approach

Next steps:

- More problems.
- Tuning. Exchange rate discovery; dynamic allocation between feasible and infeasible.
- Multi-population GAs.
- Broader comparison, e.g., with other metaheuristics.
- Mathematical and empirical analysis.